It's Elementary

A Monthly Column by EFAP Director John Yinger
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Production Functions and Cost Functions for Public Education

Economists study the production of a good or service using two closely related tools: production functions and cost functions. A production function shows the outputs that can be produced with various combinations of inputs. A cost function show how much it costs to produce various output levels given input prices.

These two tools are widely used in studying public education. Dozens of scholars have used education production functions to estimate the impact of a policy, such as smaller class sizes, on student performance (the output). Many other scholars have studied the cost of reaching various levels of student performance, given the wages that must be paid to attract teachers of a given quality (often called the opportunity wage).

A cost function is derived from a production function, but these two tools have different strengths and weaknesses in studying education. Production functions are ideally suited to studying the impact of school programs on performance in a sample of individual students. In this case, classroom-level inputs can often be observed and scholars can obtain a precise picture of the factors that influence student performance. Cost functions, which require information on spending but not on inputs, are ideally suited to studying educational production at the school district level. They can estimate the impact on the amount a district must spend of average district performance, the share of students in various cost categories, and the opportunity wage.

Although the properties of these tools are well known, they are sometimes misused. A recent example of this type of misuse comes from the 2007 Convention of the American Education Finance Association, which took place in Baltimore in March. One session at this conference featured a debate between Michael Rebell, Director of the Campaign for Educational Equity, Teachers College, Columbia

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University, and Eric Hanushek, a well known education scholar who is has a position at the Hoover Institute at Stanford University, on "The Role of the Courts in Legislating Adequacy."

During this debate Dr. Hanushek contrasted results from the cost and production function approaches using data on school districts in Missouri. Because these two approaches give very different answers for the cost of achieving any given performance standard, Dr. Hanushek argued, it was not possible to determine such costs in any meaningful way. Mr. Rebell took a different position, namely that courts can make an informed decision about the likely cost of achieving adequate student performance.

The principal problem with Dr. Hanushek's argument is that his use of production functions was not valid. To be specific, Dr. Hanushek's production function tried to explain average student performance in a district as a function of spending per pupil in the district, not as a function of the district's inputs. This is a crucial error. The use of spending as the explanatory variable requires several unreasonable assumptions, the most important of which is that every dollar of spending per pupil is assumed to have the same impact on average student performance as any other.

According to this assumption, for example, a dollar of spending on teachers must have the same impact on student performance as a dollar of spending on classrooms or a dollar of spending on administrators.[1] This assumption is not required, of course, when individual inputs (teachers, classrooms, administrators) are directly included in the analysis, but it is required when spending is included instead of separate inputs.

Because this assumption is so unreasonable, it leads to unrealistic results. According to Dr. Hanushek's "production function" regression, the impact of a dollar of spending on student performance was close to zero, so the spending required to bring student performance up to a Missouri's standards was close to infinite. This result reflects Dr. Hanushek's error, not the reality of the situation.

Dr. Hanushek also presented a cost function analysis, which implied that adequacy could be achieved with a tiny increase in spending. Based on the contrasting results from his "production function" and cost function analyses, he then concluded that reasonable estimates of the cost of achieving adequacy

[1] One (absurd) implication of this assumption is that all combinations of inputs that cost the same amount, from all teachers and no classrooms to all classrooms and no teachers, are equally productive.

were impossible to obtain. In fact, however, his cost function was mis-specified, with far fewer explanatory variables than cost functions in scholarly journals. A more complete cost function regression for Missouri by two colleagues and me provides more reasonable estimates the cost of achieving adequacy—costs that are in line with previous studies.[2]

In short, Professor Hanushek's presentation at AEFA was based on a production function that required unreasonable assumptions and a mis-specified cost function. Professor Hanushek's inappropriate use of these tools is puzzling because he has been a pioneer in developing education production functions using student-level data, which do not require unreasonable assumptions.[3] I wish that the rigor he has demonstrated in his work with student-level data had appeared in his presentation at AEFA, and I hope nobody in the audience that day was fooled by his sleight of hand.

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[2] A. Lukemeyer, W. Duncombe, and J. Yinger, "The No Child Left Behind Act: Have Federal Funds Been Left Behind?", which is forthcoming in *Public Finance Review*.

[3] See, for example, Steven G. Rivkin, Eric A. Hanushek, and John F. Kain, "Teachers, Schools, and Academic Achievement," *Econometrica*, Vol. 73, No. 2 (March, 2005), pp. 417–458.